University of Mannheim
School of Social Sciences
Mathematics for Political Scientists, Fall 2022
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Problem Set: Linear Algebra I

1. Consider the following matrices and vectors.

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 8 & 3 \\ 0 & -1 & 6 \end{pmatrix}; \ \mathbf{B} = \begin{pmatrix} -3 & 2 & 4 \\ 2 & 3 & 4 \\ 2 & -4 & 0 \end{pmatrix}; \ \mathbf{c} = \begin{pmatrix} 4 & -3 & 2 \end{pmatrix}; \ \mathbf{d} = \begin{pmatrix} 3 & 8 \end{pmatrix};$$
$$\mathbf{e} = \begin{pmatrix} 2 & 6 & 9 \end{pmatrix}; \ \mathbf{F} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}; \ \mathbf{G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \ \mathbf{H} = \begin{pmatrix} 5 & 6 & 1 \\ -2 & 7 & 8 \end{pmatrix};$$
$$\mathbf{K} = \begin{pmatrix} a_1 & ... & a_n \\ b_1 & ... & b_n \end{pmatrix}$$

   Do the calculations if possible.

   (a) $\mathbf{M}_1 = \mathbf{A} \cdot \mathbf{B}$
   (b) $\mathbf{M}_2 = \mathbf{A} - \mathbf{B}$
   (c) $\mathbf{M}_3 = \mathbf{B} \cdot \mathbf{F}$
   (d) $\mathbf{M}_4 = \mathbf{A} \cdot \mathbf{c}$
   (e) $\mathbf{M}_5 = \mathbf{c} \cdot \mathbf{A}$
   (f) $\mathbf{m}_6 = \mathbf{d} \cdot \mathbf{c}$
   (g) $\mathbf{m}_7 = 2\mathbf{c} \cdot 3\mathbf{e}$
   (h) $\mathbf{M}_8 = \mathbf{B} \cdot \mathbf{G}$
   (i) $\mathbf{M}_9 = \mathbf{A} \cdot \mathbf{H}$
   (j) $\mathbf{M}_{10} = \mathbf{H}' \cdot \mathbf{F}$

2. What is the size of the following matrices?

   (a) $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{H}'$
   (b) $\mathbf{c} + \mathbf{e} \cdot \mathbf{H}'$
   (c) $\mathbf{F} \cdot \mathbf{K}$

3. Specify whether the following matrices are square, zero, identity, diagonal or upper/lower triangular matrices and give their dimension as well as their rank.

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 & 0 & 8 \\ 0 & 1 & -2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 & 0 & 6 \\ 0 & 7 & 0 \\ 1 & -3 & 9 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 8 & 0 \\ 0 & -5 & 0 \end{pmatrix}$$

4. Is the equation $(\mathbf{F} + \mathbf{G})^2 = \mathbf{F}^2 + 2 \cdot \mathbf{F} \cdot \mathbf{G} + \mathbf{G}^2$ true for any square matrices of the same dimension?

5. Find all 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2$ is the matrix obtained from $\mathbf{A}$ by squaring each entry.