University of Mannheim
School of Social Sciences
Mathematics for Political Scientists, Fall 2022
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Problem Set: Analysis I

1. Solve the following equations.

   (a) $x^2 - 6x + 8 = 0$

   (b) $(3x-1)^2 - (5x-3)^2 = -(4x-2)^2$

   (c) $\sqrt{x^2-9} = 9 - x$

   (d) $\log_x(2x+8) = 2$

   (e) $e^{2x-5} + 1 = 4$

   (f) $\log_2 \dfrac{2}{x} = 3 + \log_2 x$, where $x > 0$

   (g) $(27)^{2x+1} = \dfrac{1}{3}$

2. Simplify the following expressions.

   (a) $\dfrac{4^2 \cdot 6^2}{3^3 \cdot 2^3}$

   (b) $\dfrac{(x+1)^3(x+1)^{-2}}{(x+1)^2(x+1)^{-3}}$

   (c) $(-3xy^2)^3$

   (d) $\dfrac{\dfrac{(x^2)^3}{x^4}}{\left(\dfrac{x^3}{(x^3)^2}\right)^{-2}}$

   (e) $[(2x+1)(2x-1)]\,(4x^2+1)$

   (f) $\dfrac{6x^5 + 4x^3 - 1}{2x^2}$

   (g) $\dfrac{1 + 4x^2 + 6x}{2x - 1}$

   (h) $\dfrac{x^2 - 5x + 4}{x^2 + 2x - 3} - \dfrac{x^2 + 2x}{x^2 + 5x + 6}$

3. Show that:

   (a) $\sum_{i=1}^{N}(x_i - \mu_x)^2 = \sum_{i=1}^{N} x_i^2 - N\mu_x^2$. Hint: Note that $\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$.
   (b) $\sum_{i=1}^{n}(a_{i+1} - a_i) = a_{n+1} - a_1$.

4. Show that $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

5. Differentiate the following functions with respect to x.

   (a) $f(x) = 7x^3 - 2x^2 - 5x + 1$
   (b) $f(x) = 0.7x^{-4} + 1.3 - 3.1x^3$
   (c) $f(x) = \dfrac{3x^2 + 1}{2x}$
   (d) $f(x) = \sqrt{4x + 9}$
   (e) $f(x) = \frac{x^{\frac{1}{3}} - 2}{(x^5 - 2)^3}$
   (f) $f(x) = \ln\left(\dfrac{x^2}{x^4 + 1}\right)$
   (g) $f(x) = e^{x^3 + x}$
   (h) $f(x) = \frac{1}{e^x + e^{-x}}$

6. Find the all first and second (mixed) partial derivatives of the following functions.

   (a) $f(x, y) = \ln x \cdot y^2$
   (b) $f(x, y) = \sqrt{2x - y}$
   (c) $f(x, y) = (x + 4y)(e^{-2x} + e^{-3y})$

7. For what value of a is the following function continuous for all x? Is it also differentiable for all x for this value of a?

$$f(x) = \begin{cases} ax - 1 & \text{if } x \leq 1 \\ 3x^2 + 1 & \text{if } x > 1 \end{cases}$$