University of Mannheim School of Social Sciences Mathematics for Political Scientists, Fall 2022 Carlos Gueiros

## Problem Set: Analysis II

- 1. Suppose the function f is defined for all  $x \in [-1.5, 2.5]$  by  $f(x) = x^5 5x^3$ .
  - (a) Determine for which values of x the value of the function is equal to zero.
  - (b) Calculate f'(x) and find the extreme points of f. What is the maximum/the minimum of the function?
  - (c) Does the function have inflection points?
- 2. Which of the following functions of x are convex? Which are concave?
  - (a)  $f(x) = (2x 1)^6$
  - (b) f(x) = 5x + 7
  - (c)  $f(x) = x^5$
  - (d)  $f(x) = \sqrt{1 + x^2}$
  - (e)  $f(x) = x^5$  for  $x \ge 0$
  - (f)  $f(x) = 5x^2 x^4$  for  $x \ge 1$
- 3. Appeasement Problem (Ashworth and Bueno de Mesquita, 2006)

Two states must divide some territory. There is a status quo division, but one state (call is D) is dissatisfied with the status quo. The other state (call is S) is satisfied with the status quo division. S gets one chance to try to appease D by offering it some of the disputed territory. Let x be the fraction of S's territory that it offers. S is uncertain about how dissatisfied D is. S believes that D will accept an offer of x with probability p(x) = x. If D accepts the offer, then war is averted and S is left with 1 - x of its territory. If D rejects the offer, then there is a war. S believes that it will win a war with probability q. Thus, q can be thought of as S's relative military strength. If S wins the war, then S keeps all of its territory. If S looses the war, it ends up with none of the disputed territory.

Given all of this, S's maximization problem is given by

$$\max_{x}(1-x)x + q(1-x)$$

- (a) Find the solution to S's maximization problem.
- (b) How does the level of appeasement, x, changes with S's perception of its military strength?
- 4. A government has to decide about the allocation of its budget. Let x denote the share of the budget used for military and y the share of the budget used for social expenditures. The government has to use of all its budget and has the following utility function:

$$u(x,y) = e^{2x} + e^{2y}$$

Solve the government's optimization problem.

- 5. Consider the function  $f(x) = (x^2 + 2x)e^{-x}$ .
  - (a) Determine for which values of x the value of the function is equal to zero.
  - (b) Calculate f'(x) and find the extreme points of f. What is the maximum/the minimum of the function?
  - (c) Does the function have inflection points?
  - (d) Sketch the function and specify whether it is convex/concave (in sections).
- 6. Solve the indefinite integrals:

(a) 
$$\int \frac{1}{\sqrt{x}} dx$$
  
(b) 
$$\int e^{-4t} dt$$
  
(c) 
$$\int x \sqrt{x} dx$$
  
(d) 
$$\int \frac{1}{x} dx$$
  
(e) 
$$\int (2x^2 + x - 3) dx$$
  
(f) 
$$\int \frac{(x^4 + 1)^2}{x^3} dx$$

7. Calculate  $\int_0^2 (2x^2 + x - 3) dx$ . Hint: Make a sketch of the function before.