University of Mannheim School of Social Sciences Mathematics for Political Scientists, Fall 2022 Carlos Gueiros

## Solutions: Probability Theory II

- 1. Consider the p.d.f. f(x) = 2x for  $0 \le x \le 1$ .
  - (a) Calculate the c.d.f. of f(x).

$$F(x) = x^2$$

(b) Is f(x) a proper p.d.f.?

$$\int_0^1 F(x) = 1$$
$$1^2 - 0^2 = 1$$

f(x) is a proper p.d.f.

- 2. Consider the c.d.f.  $G(x) = \frac{1}{9}x^2$  for  $0 \le x \le 3$ .
  - (a) Calculate the p.d.f. of G(x), g(x).

$$g(x) = \frac{2}{9}x$$

(b) Is g(x) a proper p.d.f.?

$$\int_{0}^{3} G(x) = 1$$
$$\frac{1}{9} \times 3^{2} - \frac{1}{9} \times 0^{2} = 1$$

g(x) is a proper p.d.f.

3. Consider the p.d.f.  $h(x) = \frac{4}{3}(1-x^3)$  for 0 < x < 1. Determine

(a)  $\Pr(X < \frac{1}{2}).$ 

$$H(x) = -\frac{1}{3}x(x^3 - 4)$$
$$H\left(\frac{1}{2}\right) \approx 0.65$$

(b)  $\Pr(X > \frac{1}{3}).$ 

$$1 - H\left(\frac{1}{3}\right) \approx 0.56$$

(c)  $\Pr(\frac{1}{4} < X < \frac{3}{4}).$ 

$$H\left(\frac{3}{4}\right) - H\left(\frac{1}{4}\right) \approx 0.56$$

- 4. Consider the p.d.f.  $k(x) = cx^2$  for  $1 \le x \le 2$ . Determine
  - (a) Find the value of the constant c.

$$\int_{1}^{2} k(x)dx = 1$$
$$\frac{1}{3}cx^{3}\Big|_{1}^{2} = 1$$
$$c = \frac{3}{7}$$

(b) Find  $\Pr(X > \frac{3}{2})$ .

$$K(x) = \frac{1}{7}x^3$$
$$1 - K\left(\frac{3}{2}\right) \approx 0.52$$